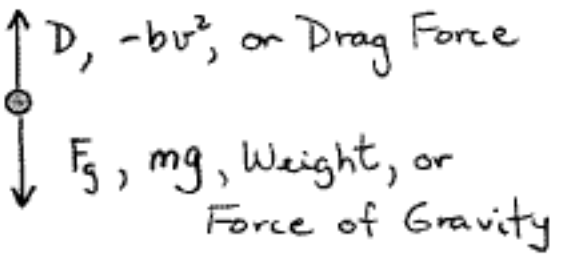
2000 Physics C Solutions **Distribution of points**

Mech. 2 (15 points)

(a) 3 points



For a vector arrow pointing downward — 1 point
For a vector arrow pointing upward — 1 point
For correct force labels on both vectors — 1 point
For any extra vectors drawn, deduct 1 point

(b) 3 points

For indicating that the acceleration decreases — 1 point
For a correct explanation that includes a correct mention of forces. — 2 points
For example, as the ball approaches terminal speed, the velocity increases, so the drag force increases and gets closer in magnitude to the gravitational force. The resultant force, which is the difference between the gravitational and drag forces, gets smaller, and since it is proportional to the acceleration, the acceleration decreases.

Partial credit of 1 point given for only a statement including a basic definition of terminal velocity (e.g., at terminal velocity v = constant, so a must decrease from 9.8 m/s^2 to zero)

(c) 2 points

For an expression for the resultant force on the ball — 1 point

$F = mg - bv^2$

Since $F = ma = m\frac{dv}{dt}$, then $m\frac{dv}{dt} = mg - bv^2$

For a correct differential equation — 1 point

$\frac{dv}{dt} = g - \frac{b}{m}v^2$

Students did not need to use the convention + and – for up and down, respectively, but they did have to be consistent in their sign notation for credit. The integral form of the differential equation was also acceptable.

Mech. 2 (continued)

(d) 3 points

For recognition that acceleration is zero at terminal speed — 1 point
For setting the drag force equal to the gravitational force — 1 point
$mg = bv_t^2$
For a correct solution for v_t — 1 point
$v_t = \sqrt{\frac{mg}{b}}$

Full credit also given for writing answer only with no other work shown

(e) 4 points

For a correct statement of work-energy, recognizing that the energy dissipated by the drag force is equal to the initial energy minus the final energy — 1 point
For correct recognition of both initial potential energy mgh and final kinetic energy $\frac{1}{2}mv_t^2$ — 1 point
$\Delta E = mgh - \frac{1}{2}mv_t^2$
For correct substitution of v_t from part (d) — 1 point
$\Delta E = mgh - \frac{1}{2}m\left(\frac{mg}{b}\right)$
For correct answer — 1 point
$\Delta E = mg\left(h - \frac{m}{2b}\right)$

Alternate partial solution (for maximum credit of 2 points) — *(Alternate points)*
For a correct integral for work — *(1 point)*
$W = \int P\,dt$ OR $W = \int F\,dx$
For correct substitutions for P or F — *(1 point)*
$W = \int bv^3\,dt$ OR $W = \int bv^2\,dx$ OR $W = \int kv^2\,dx$

AP® PHYSICS C MECHANICS
2005 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**

(a) 2 points

For indicating that the magnitude of the acceleration decreases as the ball moves upward 1 point
For a correct, reasonable justification 1 point
For example: Since velocity is upward, air resistance is downward, in the same direction as gravity. The velocity will decrease, causing the force of air resistance to decrease. Therefore, the net force and thus the total acceleration both decrease.

(b) 3 points

For showing the expression $a = \frac{dv}{dt}$ 1 point

For any clear indication that the forces of air resistance and gravity are in the same direction, such as by showing an equation or a free-body diagram 1 point

$F_{\text{net}} = -Mg - kv$

For a correct differential equation with the correct signs 1 point

$M\frac{dv}{dt} = -Mg - kv$

(c) 3 points

For recognizing that at terminal speed $F_{\text{net}} = 0$ 1 point

For any clear indication that the forces of air resistance and gravity are now in opposite directions, such as by showing an equation or a free-body diagram 1 point

$F_{\text{net}} = -Mg + kv$

$0 = -Mg + kv_{\text{T}}$

For a correct expression for the terminal speed 1 point

$v_{\text{T}} = Mg/k$

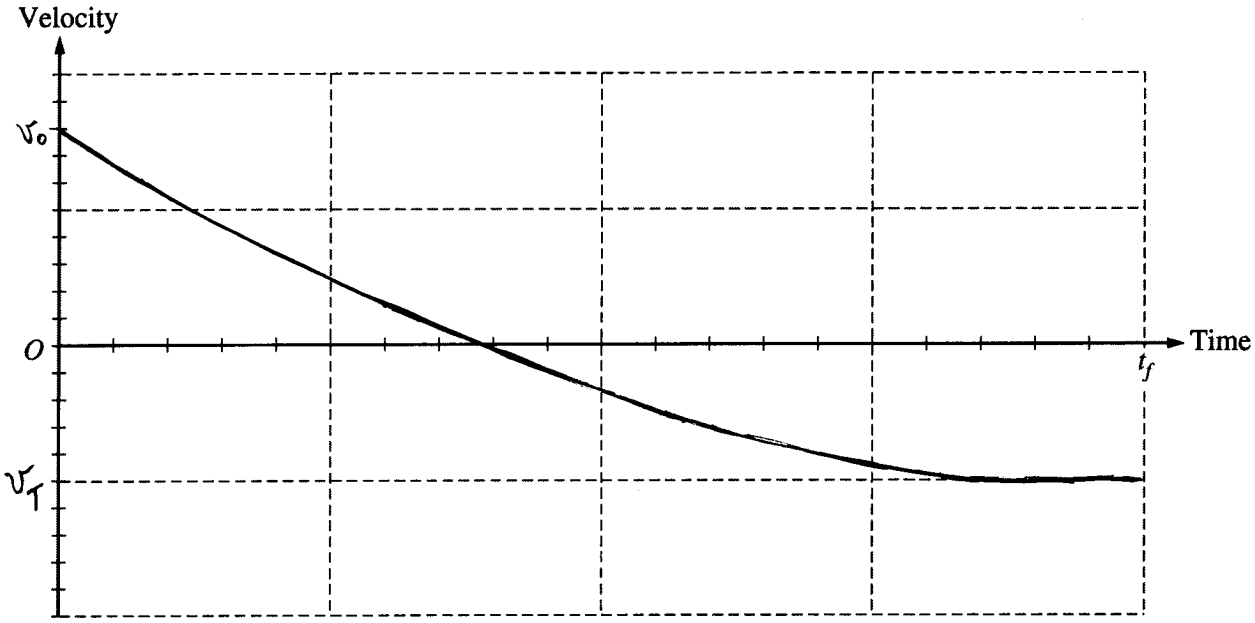
(d) 2 points

For indicating that it takes longer for the ball to fall 1 point
For a correct, reasonable justification 1 point
For example: The ball loses mechanical energy on the way up and on the way down. This means the average speed is greater on the way up than on the way down. Since the distance traveled is the same, the time must be longer on the way down.

Question 1 (continued)

Distribution of points

(e) 5 points



For an exponentially decreasing curve beginning with positive initial velocity v_0, crossing the time axis at t less than $t_f/2$, and having the final speed less than the initial speed — 5 points

One point partial credit was awarded for each of the following curve characteristics.

For showing that when $v = 0$, the curve is differentiable (i.e., no discontinuity in slope) and has a negative slope

For showing the curve to be concave upward both for when the ball is rising and when the ball is falling

For showing time intervals for when the ball is rising and when the ball is falling that are consistent with the answer to part (d)

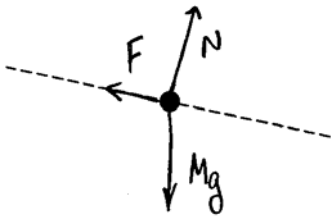
For showing that the final velocity is negative and that the speed at t_f is less than v_0

AP® PHYSICS C: MECHANICS
2008 SCORING GUIDELINES

Question 1

15 points total **Distribution of points**

(a) 3 points



For a correctly drawn and labeled weight vector, originating on the dot and with an arrowhead (Alternatively, correctly drawn and labeled components instead of the total weight vector was acceptable.) 1 point

For a correctly drawn and labeled normal force vector, originating on the dot and with an arrowhead 1 point

For a correctly drawn and labeled drag-force vector, originating on the dot and with an arrowhead 1 point

One point was deducted if there were any extra vectors on the point, including components drawn with arrowheads.

(b) 4 points

For any expression of $F = Ma$ or any dimensionally correct application of $F = Ma$ 1 point

For correctly expressing the component of the weight parallel to the plane as $Mg\sin\theta$ 1 point

For correctly expressing the drag force as $-kx$ 1 point

$Ma = Mg\sin\theta - b\upsilon$

For a dimensionally correct differential equation, including $d\upsilon/dt$ and expressions for the drag force and the component of the weight parallel to the plane 1 point

$$M\frac{d\upsilon}{dt} = Mg\sin\theta - b\upsilon$$

One point was deducted if the algebraic signs of the weight component and the drag force were not opposite somewhere in the solution, OR if only one of these two terms was included.

(c) 2 points

For an indication that $F_{\text{net}} = 0$, $a = 0$, or the parallel component of the weight = $b\upsilon_T$ 1 point

$0 = Mg\sin\theta - b\upsilon_T$

$b\upsilon_T = Mg\sin\theta$

For the correct expression for the terminal velocity (or one consistent with part (b)) 1 point

$\upsilon_T = Mg\sin\theta/b$

Question 1 (continued)

Distribution of points

(d) 3 points

For taking the differential equation from part (b) and correctly separating the variables in preparation for integration (definite or indefinite integral) — 1 point

$$M\frac{dv}{dt} = Mg\sin\theta - bv$$

$$\frac{dv}{Mg\sin\theta - bv} = \frac{dt}{M}$$

For correct integration of both sides of equation — 1 point

For example, using a method involving an indefinite integral

Letting $u = Mg\sin\theta - bv$, so $du = -b\,dv$

$$-\frac{1}{b}\frac{du}{u} = \frac{dt}{M}$$

$$\int\frac{du}{u} = -\frac{b}{M}\int dt$$

$$\ln u = -\frac{b}{M}t + \ln C$$

$$u = Ce^{-bt/M}$$

$$Mg\sin\theta - bv = Ce^{-bt/M}$$

Using $v = 0$ at $t = 0$

$$Mg\sin\theta = C$$

$$Mg\sin\theta - bv = Mg\sin\theta\, e^{-bt/M}$$

$$-bv = Mg\sin\theta\, e^{-bt/M} - Mg\sin\theta$$

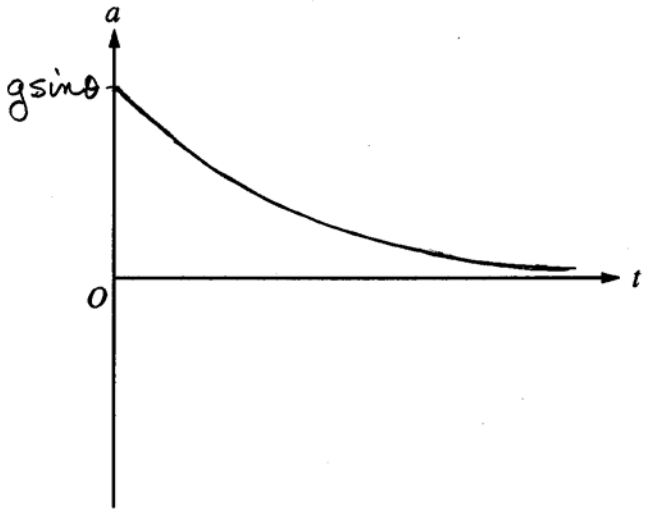
For a correct final expression for $v(t)$ — 1 point

$$v = (Mg\sin\theta/b)\left(1 - e^{-bt/M}\right)$$

Question 1 (continued)

Distribution of points

(e) 3 points



For the correct initial value of a (or a value consistent with part (b)) 1 point
For a negatively sloped curve, concave up 1 point
For a curve asymptotic to the t axis 1 point
(This point was awarded even if the curve was not otherwise correct.)

Question 2 (continued)

Distribution of points

(e)

i. 2 points

$\Sigma F = ma$

For substituting the braking force into Newton's second law as the net force — 1 point

For substituting the time derivative of velocity for the acceleration — 1 point

$-ku = M(du/dt)$

ii. 2 points

For separating the variables and integrating — 1 point

$du/u = -(k/M)\,dt$

$\int_{u_D}^{u} du/u = -(k/M)\int_{0}^{t} dt$

$\ln u\big|_{u_D}^{u} = -(k/M)t$

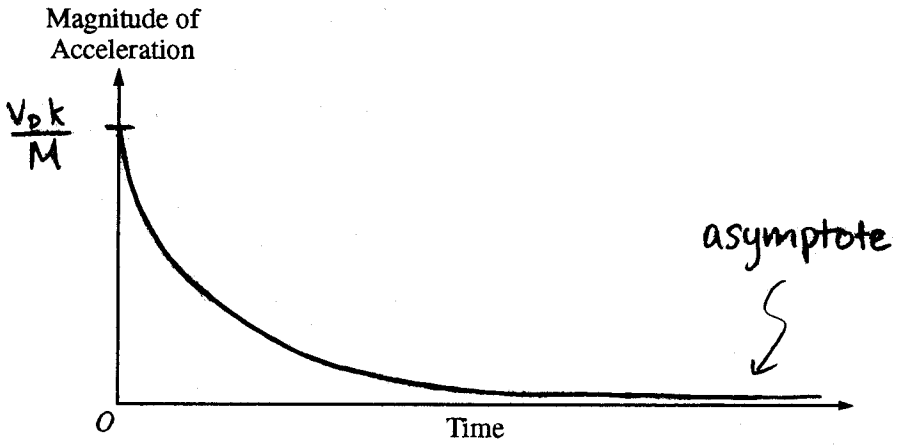
$\ln u - \ln u_D = \ln(u/u_D) = -(k/M)t$

$u/u_D = e^{-kt/M}$

For a correct expression for the velocity as a function of time — 1 point

$u = u_D e^{-kt/M}$

iii. 3 points



Taking the derivative of the equation for u from part (e) ii

$a = du/dt = d\left(u_D e^{-kt/M}\right)\big/dt = -(k/M)u_D e^{-kt/M}$

At $t = 0$, $a = -ku_D/M$

For a graph with a finite intercept on the vertical axis — 1 point

For a graph that is concave upward and asymptotic to zero — 1 point

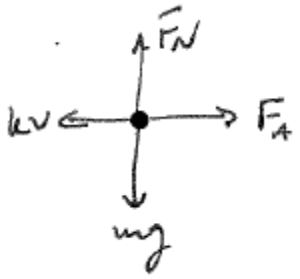
For labeling the initial acceleration with the correct value — 1 point

AP® PHYSICS C: MECHANICS
2013 SCORING GUIDELINES

Question 2

15 points total **Distribution of points**

(a) 4 points



For correctly showing and labeling the applied force directed to the right — 1 point
For correctly showing and labeling the downward gravitational force — 1 point
For correctly showing and labeling the upward normal force — 1 point
For correctly showing and labeling the drag force directed to the left — 1 point
One earned point was deducted for having any extraneous vectors

(b) 2 points

$F_{net} = ma$

For the correct substitution into Newton's second law — 1 point

$F_A - kv = ma$

For a correct differential equation — 1 point

$F_A - kv = m\frac{dv}{dt}$

(c) 1 point

Set $\frac{dv}{dt} = 0$ in the equation from part (b)

$F_A - kv = 0$

For the correct expression for the terminal velocity — 1 point

$v_T = \frac{F_A}{k}$

Question 2 (continued)

Distribution of points

(d) 5 points

Use the differential equation from part (b)

$F_A - k\upsilon = m\frac{d\upsilon}{dt}$

For demonstrating separation of variables — 1 point

$\frac{1}{m}dt = \frac{1}{F_A - k\upsilon}d\upsilon$

For demonstrating that the equation must be integrated — 1 point

$\int \frac{1}{m}dt = \int \frac{1}{F_A - k\upsilon}d\upsilon$

For demonstrating substitution using initial and final values (or evaluating the constant of integration using the boundary conditions) — 1 point

$\int_0^t \frac{1}{m}dt = \int_0^{\upsilon(t)} \frac{1}{F_A - k\upsilon}d\upsilon$

$\left[\frac{t}{m}\right]_0^t = -\frac{1}{k}\left[\ln\left(F_A - k\upsilon\right)\right]_0^{\upsilon(t)}$

For attempting to solve for $\upsilon(t)$ — 1 point

$-\frac{kt}{m} = \ln\left(\frac{F_A - k\upsilon(t)}{F_A}\right)$

$e^{-kt/m} = \frac{F_A - k\upsilon(t)}{F_A} = 1 - \frac{k\upsilon(t)}{F_A}$

$\frac{k\upsilon(t)}{F_A} = 1 - e^{-kt/m}$

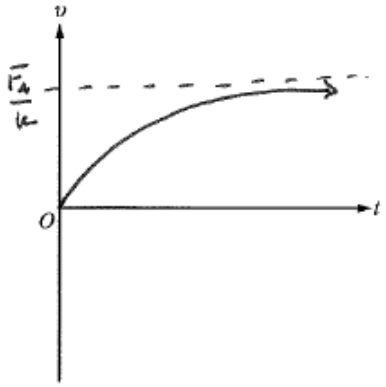
For a correct answer — 1 point

$\upsilon(t) = \frac{F_A}{k}\left(1 - e^{-kt/m}\right)$

Question 2 (continued)

Distribution of points

(e) 3 points



For a graph that begins at the origin, with a non-negative slope everywhere, and is concave downward — 1 point

For a graph with a horizontal asymptote — 1 point

For the correct label of the expression for the asymptote or maximum on the vertical axis — 1 point